Locally resonant metamaterials for plate waves: the respective roles of compressional versus flexural resonances of a dense forest of vertical rods

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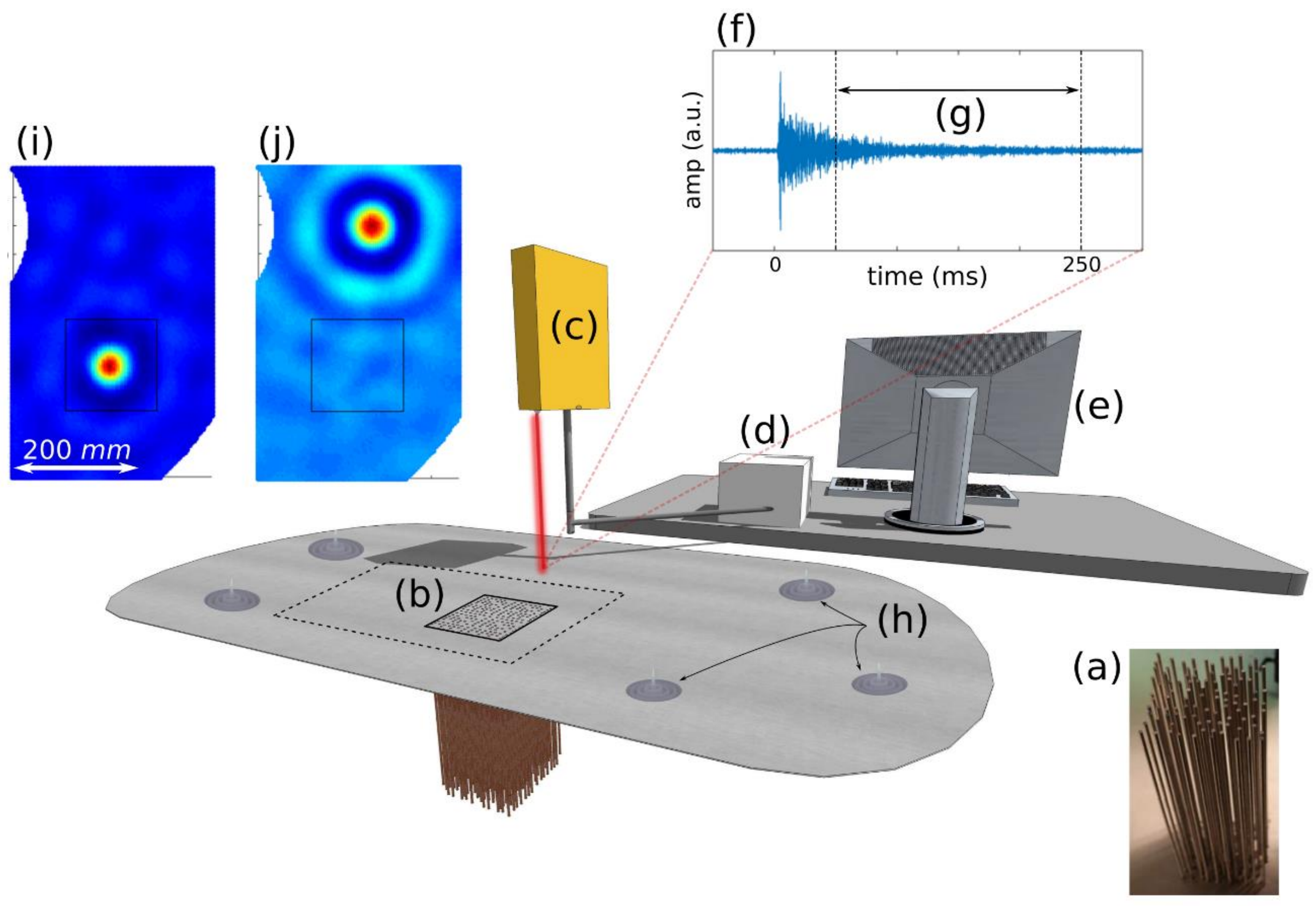
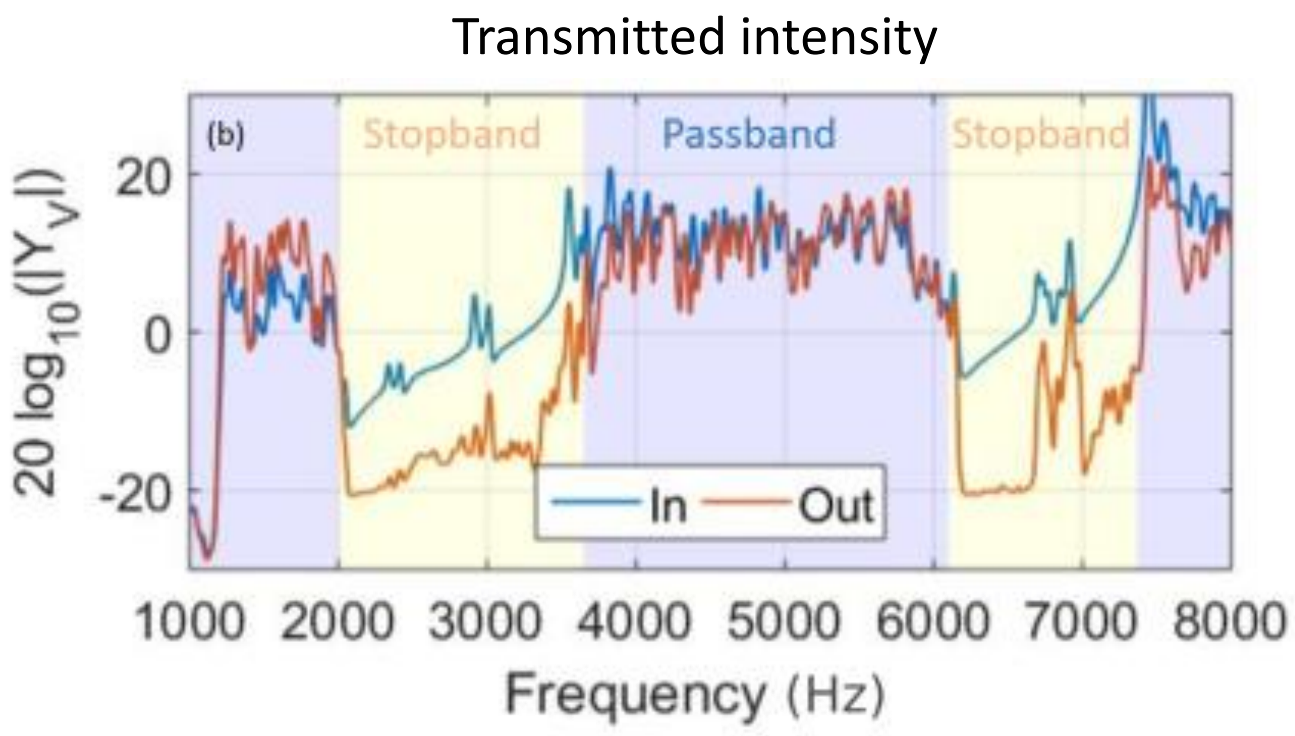
We study wave propagation phenomena observed in a mesoscopic metasurface made of a randomly-distributed set of long vertical metallic rods glued on a thin elastic plate. The main effects of the rods on the propagation of the lamb wave mode A0 is due to the low quality factor compressional resonances of the rods. At the resonance, the rods modifies the apparent stiffness of the plate. At the anti-resonance, those rods clamp the plate in the metamaterial region, which induces large frequency bandgaps. Between these two resonant and anti-resonant frequencies, a continuum values of the effective rigidity change the reflectivity and so the corresponding impedance of the metasurface in amplitude and phase.

Experimental Setup



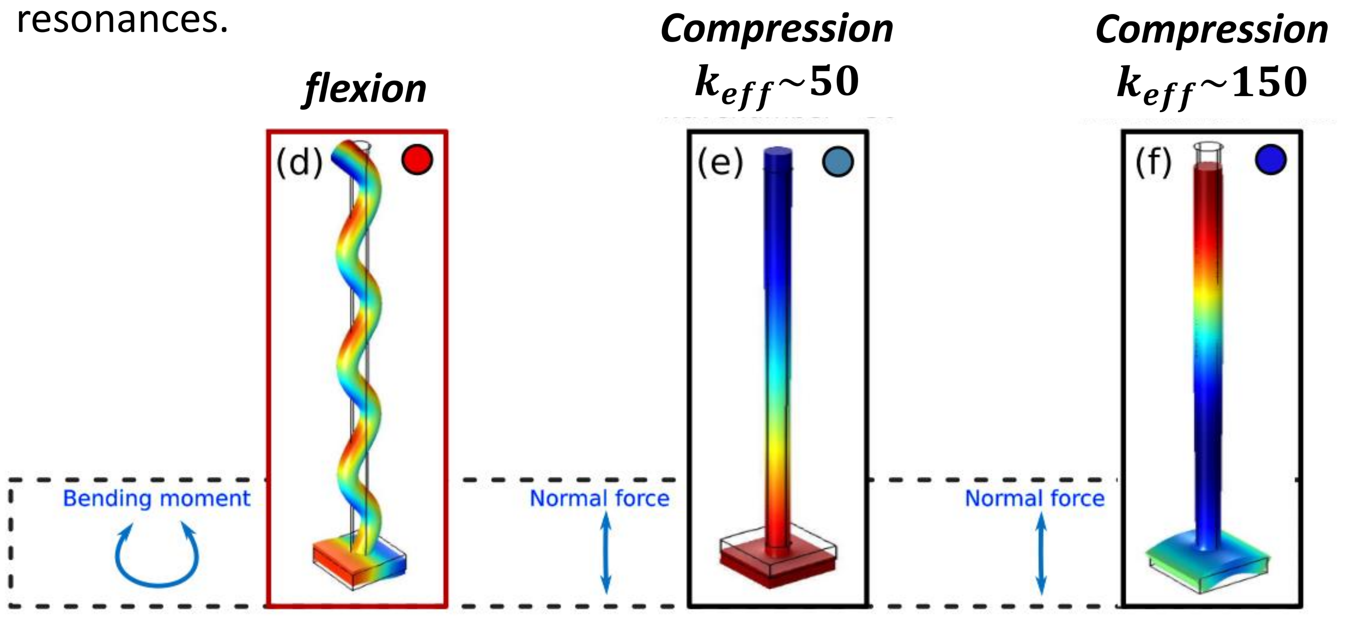
Experimental set-up at the laboratory scale.

The metamaterial **(a)** is made of 100 vertical aluminum rods glued to the opposite side of a plate in the same material (black square **(b)**). The particle velocity is measured using an out-of-plane laser Doppler velocimeter **(c)** attached to a PC-controlled **(d)** motorized robot arm **(e).** The recorded signal **(f)** is strongly reverberated due to the billiard-table shape of the plate. Using the late part coda **(g)** and few sources **(h)**, homogeneous focal spot are reconstructed from correlations inside **(i)** and outside **(j)** the beam cluster.

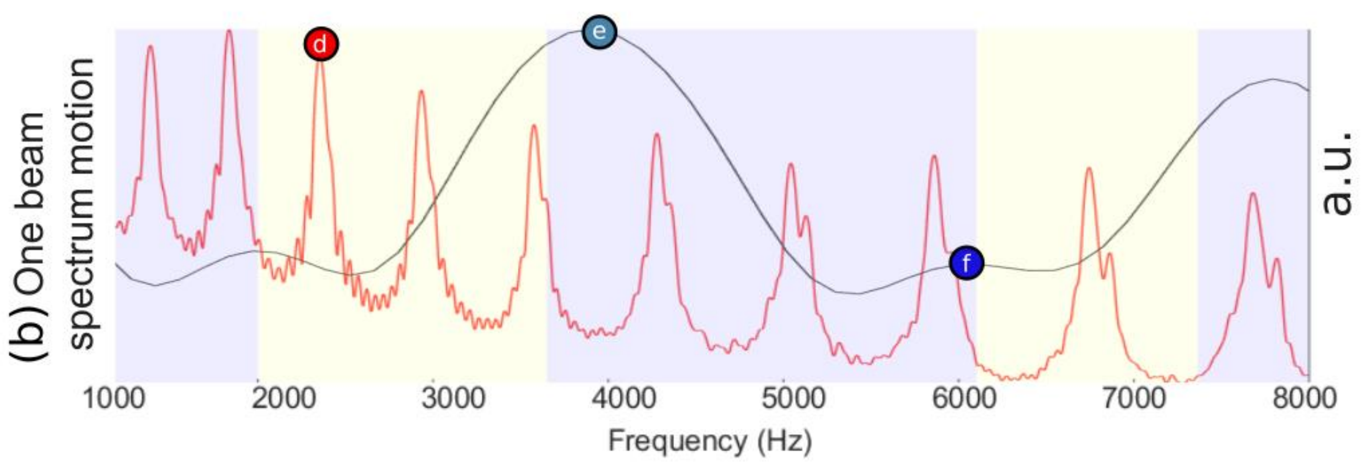


Impedance and mechanical coupling of a single rod attached to the plate

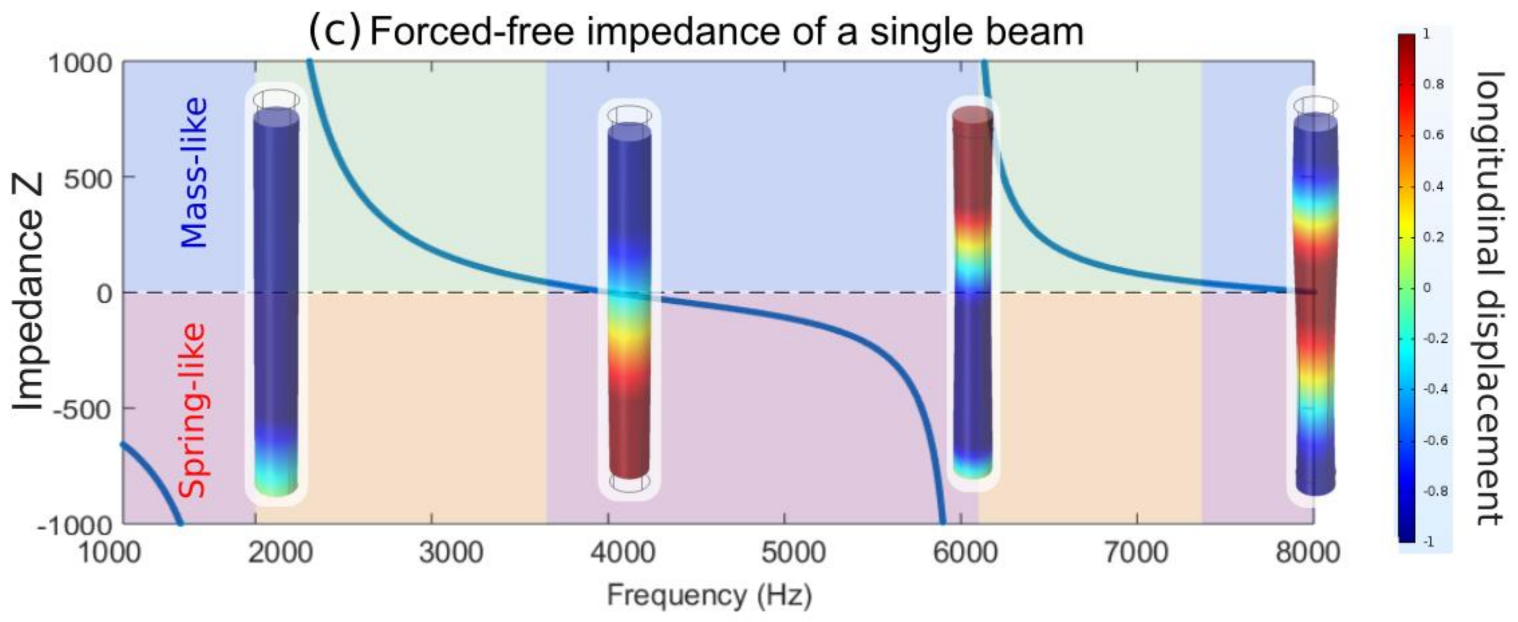
(d-f) Bloch-waves polarization obtained from COMSOL simulation for a single rod attached to a plate with periodic Bloch boundary conditions, extracted at different frequencies close to the compressional or flexural resonances.



(b) Spectrum motion of a single rod. the out-of-plan motion is shown in black, and the in-plan motion in red.
(c) Driving point impedance calculation at the base of a single rod attached to the plate, when the rod displacement is limited to longitudinal (vertical) motion

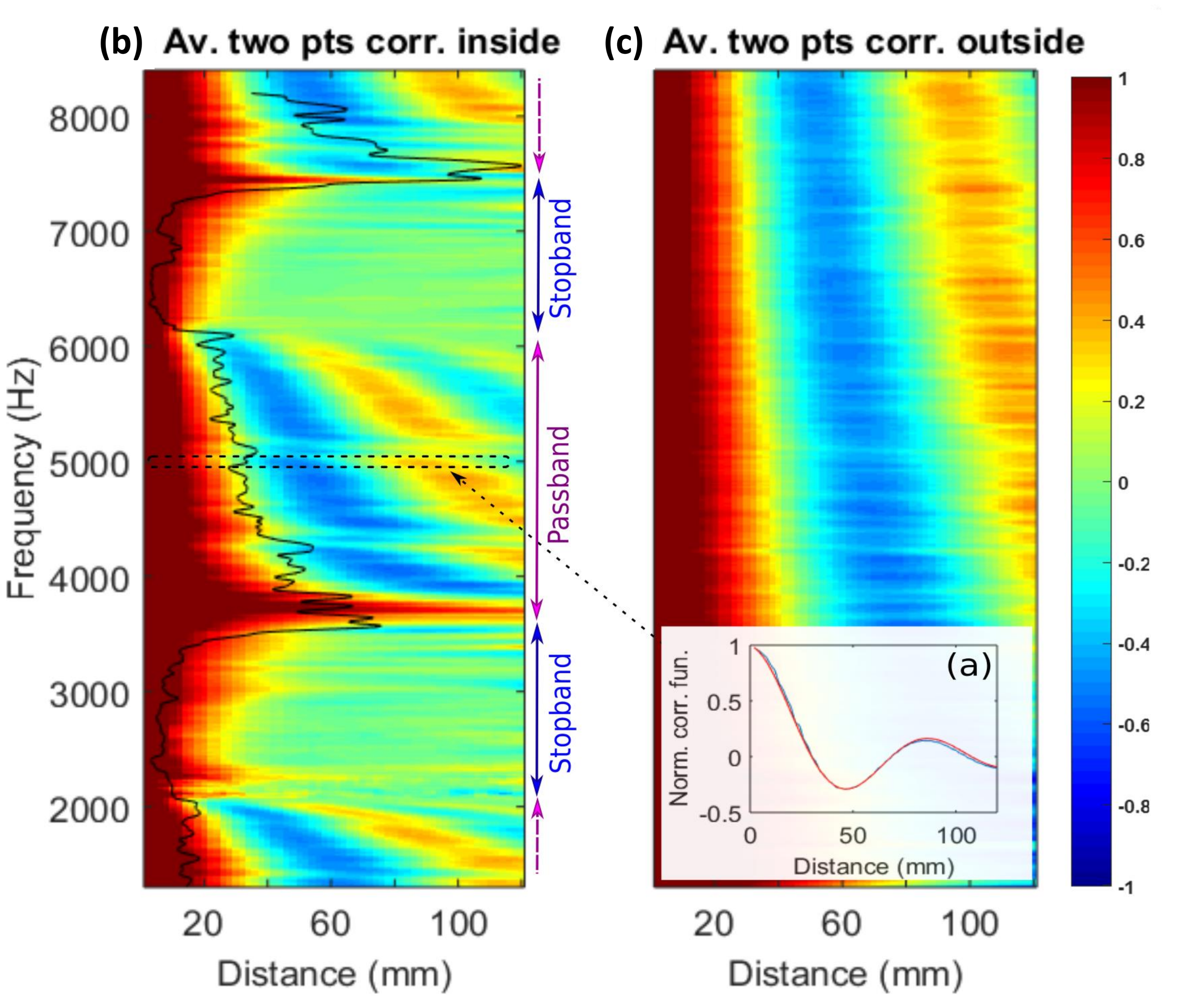


Instrumentation of a single rod for longitudinal (black arrow) and flexural (red arrow) motion.



2-pts correlation

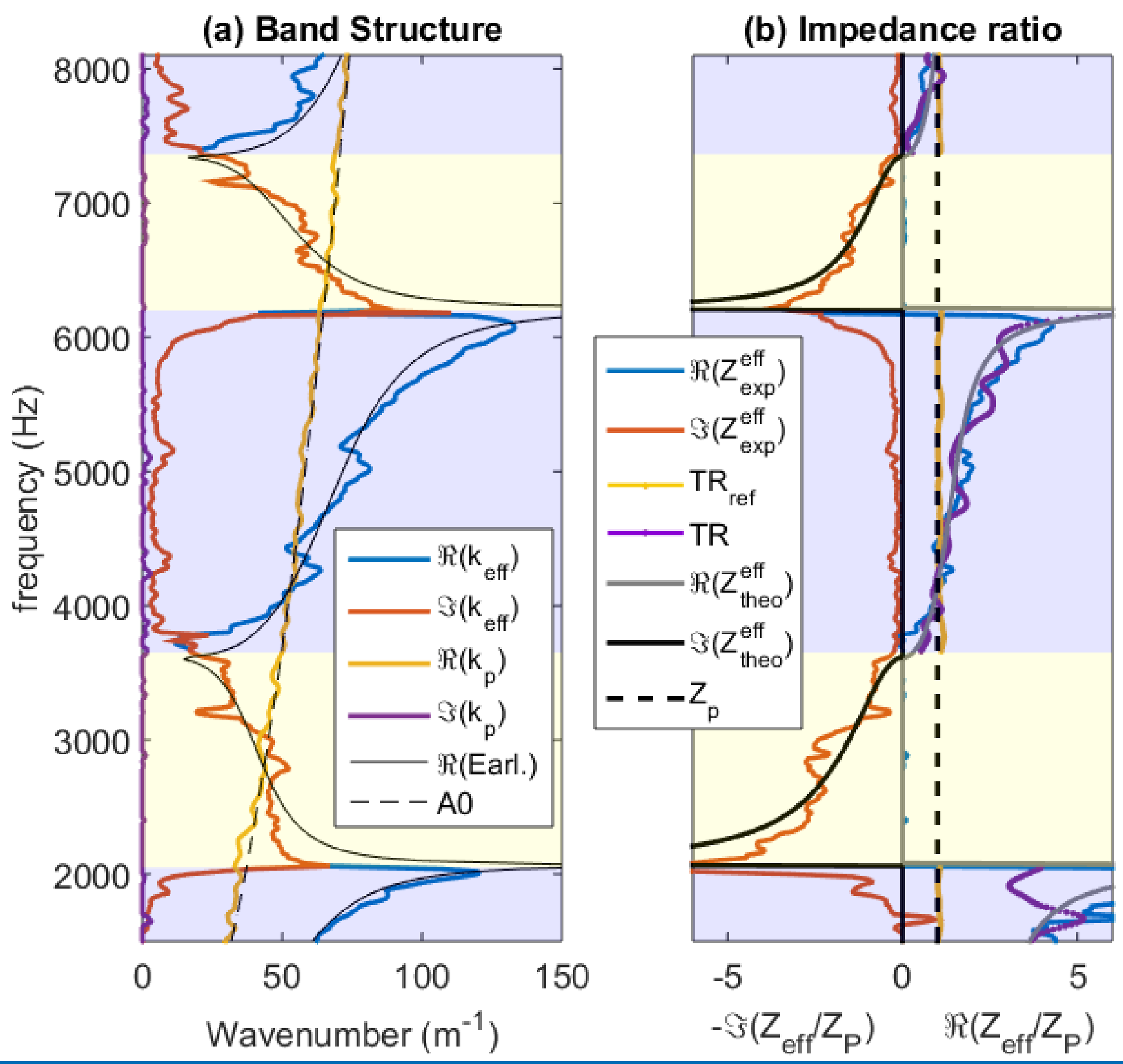
Two points correlation method : (a) Real part of the averaged two-point correlation function (normalized) measured at 5 kHz for the entire receiver pairs located inside the metamaterial region (blue). The modeled plate Greens function is plotted in red. **(b)** Averaged two-point correlation versus frequency measured inside the metamaterial **(c)** and outside the metamaterial. The black line in **(b)** corresponds to the averaged intensity versus frequency measured inside the metamaterial.



Effective velocity and attenuations

$$k_{eff} = k_p \left[\frac{M_b}{M} \frac{\tan(k_b L_b)}{k_b L_b} + 1 \right]^{1/4}$$

(1) Effective wavenumber (from [1])
k_p is the free plate wavenumber, M_b and M the mass of the beam and the portion of plate below and k_b and L_b the longitudinal wavenumber of the beam and its length. In the bandgap, k_p is complex and $\Re(k_{eff}) = \pm\Im(k_{eff})$



Metamaterial band structure. The blue and red curves are computed from the averaged two-point correlation function using the analytical Green function expression for the free plate. The black solid line depict the real part of the analytical effective wavenumber from Earl W [1]. The A0 dispersion curve is plotted in dashed line. **(b)** Relative impedance values obtained through different methods. In purple, from the time reversal focal spot amplitude (in the passband only). In blue and red from the two-point correlation method and using equation(1). In black and grey, theoretical values from Earl W. effective medium formulation and using equation **Eq.(1)**. Dashed line is the relative constant impedance value of the plate. The background colors highlight the bandwidth of the stopband (red) and passband (blue).

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